BOSTON UNIVERSITY

Discussion: Functional Partial Membership Models

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Recap

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- Functional data analysis (Ramsay and Silverman, 2005): stochastic process $f : \mathcal{T} \to \mathbb{R}$, e.g. $f \sim GP(\mu, C)$, square integrable $f \in L^2(\mathcal{T})$, can be *decomposed* as

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- Mixed membership models (Blei *et al*, 2003): for K *features* $f^{(k)}$, take the mixture

$$f = \sum_{k=1}^{K} Z_k f^{(k)}$$

where the indicators $Z_k \in (0, 1)$ with $\sum_{k=1}^{K} Z_k = 1$

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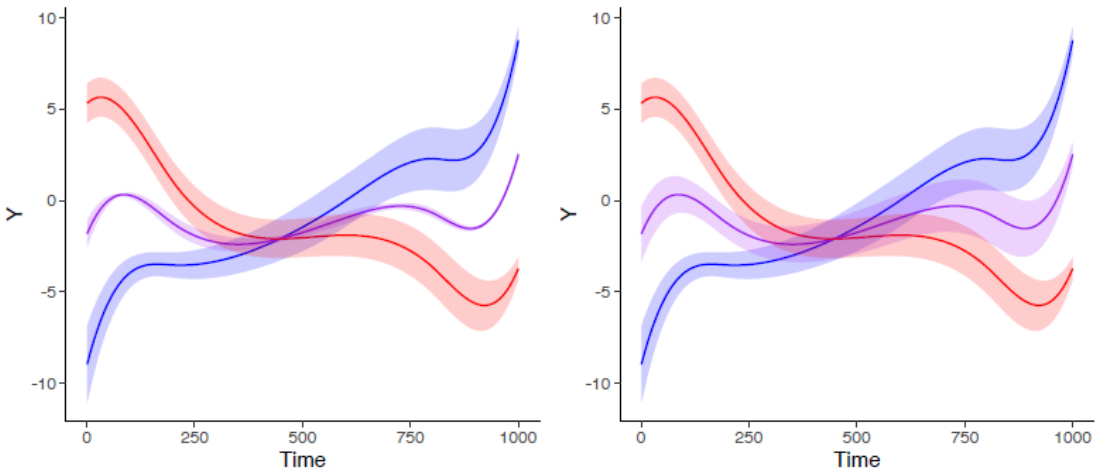

Questions

- **Non-identifiability**: interesting points were raised, but not fully explored
    - Label switching: details on algorithm under mixed membership? Can a prior on the label configuration space be used instead? Ideal for an objective prior!
    - Orthogonality seems to be essential for identifiability, but it was lifted for computational feasibility in conjunction with the MGPSP. What are the trade-offs? Were they explored?

Questions

- **Model fine-tuning and guarantees**:
  - How constraining is the use of the MGPSP in modeling the cross-covariance structure? How are the hyper-parameters specified?
  - How are the dimensions --- K (number of features), P (basis rank), and M (covariance rank) --- defined in practice? Information criteria for K might not be very reliable and overestimate in practice, so maybe another good opportunity for an objective prior!

Questions

- **Implementation**:
  - How feasible is the computation here? Metropolis-within-Gibbs seems to lead to long convergence times, especially under non-identifiability!
  - How is the tempering schedule calibrated in practice?